

光センサーの重み付け最適化による MEG II 実験ガンマ線エネルギー再構成の改善

Improvement of gamma-ray energy reconstruction of the MEG II experiment by optimizing the weighting of photosensors

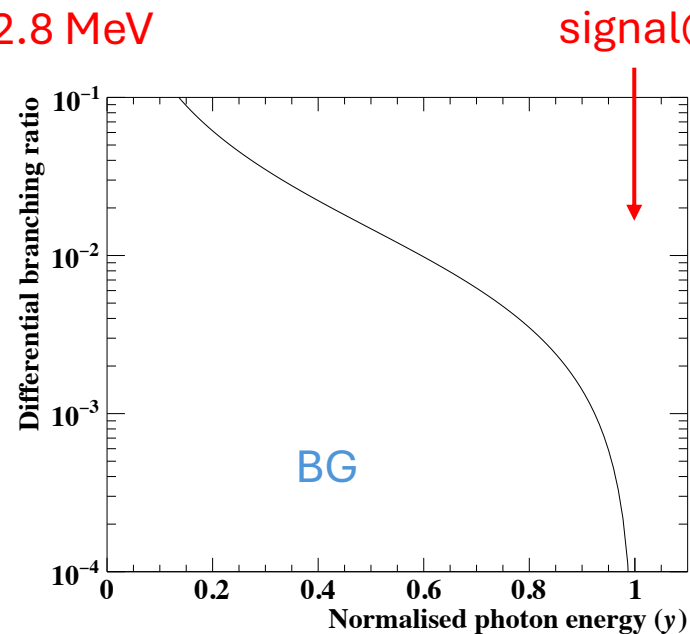
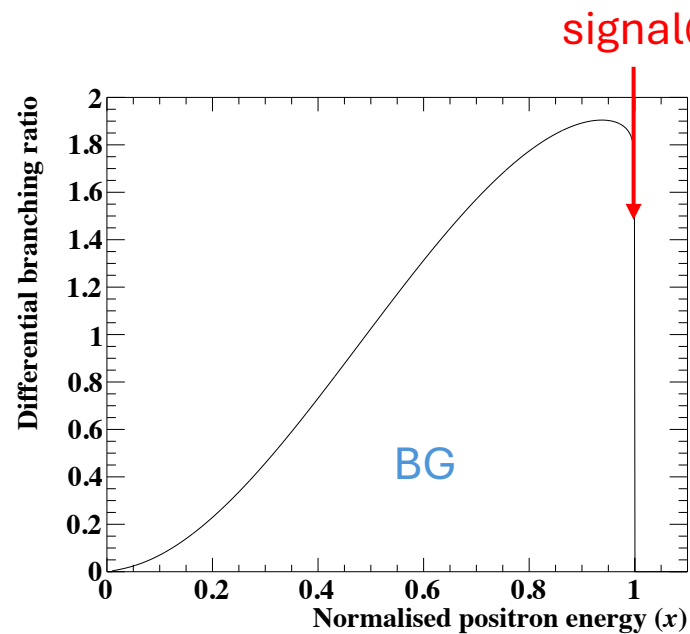
Weiyuan Li (UTokyo) on behalf of the MEG II Collaboration



E_γ Resolution Matters!

Background:

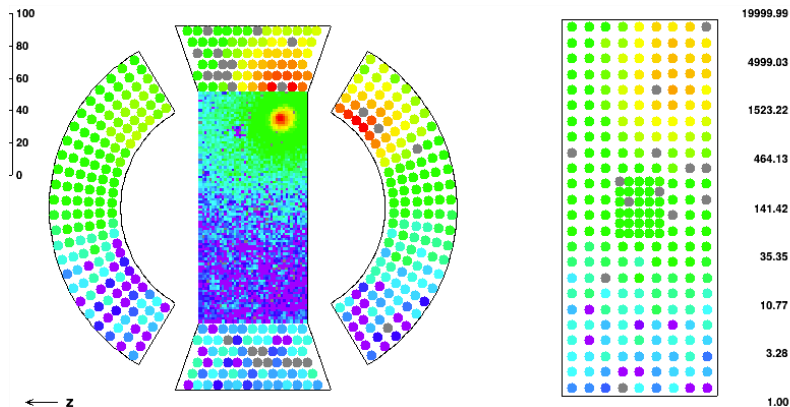
$$N_{\text{acc}} \propto R_\mu^2 \cdot \delta E_{e^+} \cdot (\delta E_\gamma)^2 \cdot \delta \Theta_{e^+\gamma}^2 \cdot \delta t_{e^+\gamma}$$



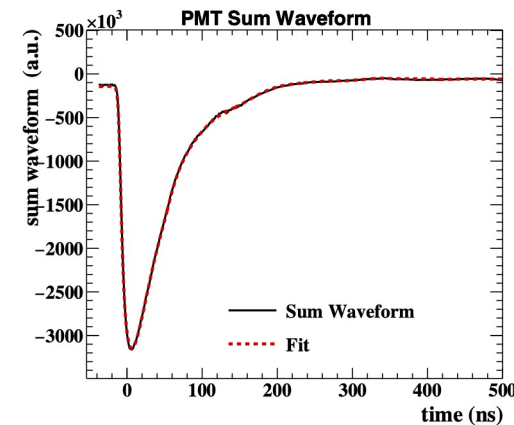
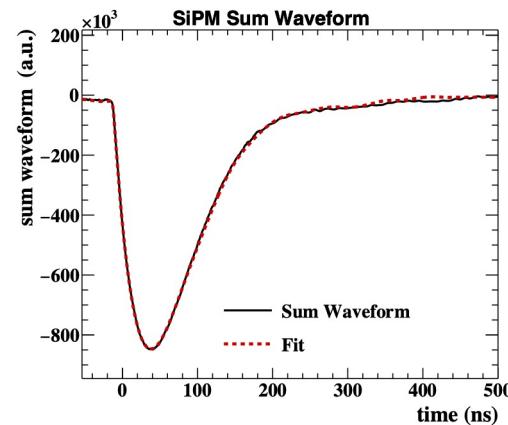
E_γ resolution makes difference

Current E_γ Reconstruction

LXe detector calibrated
4092 SiPMs + 668 PMTs



Sum waveform + template fit



E_γ

per-face weights (6 factors)
depending on the position of gamma-ray

→ **optimize for the better E_γ resolution**

Per-sensor Weight Optimization

$$E^{\text{LXe}} = \sum_{i \in PM} w_i N_i$$

energy deposit in LXe

weight for i -th sensor

#photons detected by i -th sensor

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→ How can we obtain the optimal weights?

Least Square Fit

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Least Square Fit

Use γ source
whose energy is well known

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- 55 MeV γ from $\pi^- p \rightarrow \pi^0 n, \pi^0 \rightarrow \gamma\gamma$
*The energy depends on the opening angle of two γ
- 17.6 MeV γ from ${}^7\text{Li}(p, \gamma){}^8\text{Be}$

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Collect enough events

$$E_j^{\text{ref}} = \sum_{i \in PM} w_i N_{i,j} \quad j = 1, \dots, N_{\text{events}} (\gg N_{PM})$$

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In practice, we solve $Aw = b$ where, $A = N^T N, b = N^T e^{\text{ref}}$

Regularization

$$Aw = b$$



Aha, looks easy!

Regularization

$$\mathbf{w} = \mathbf{A}^{-1}\mathbf{b}$$



Collinearity!
Noise!
Negative weights!

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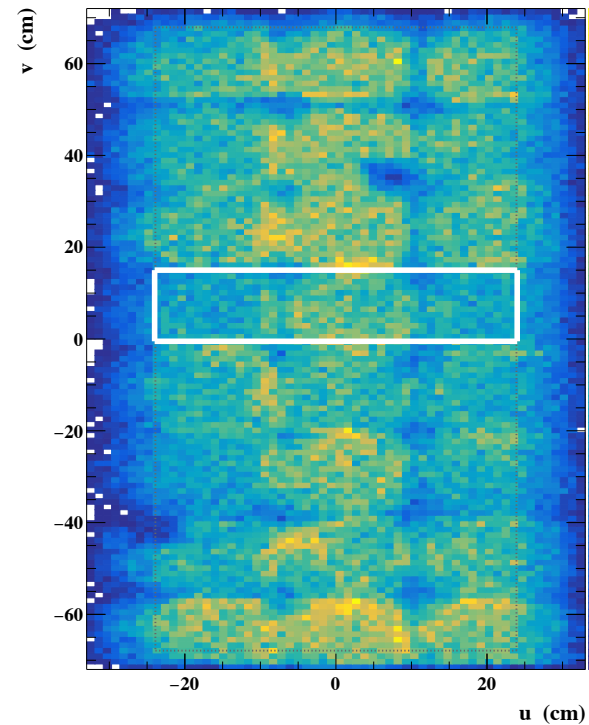


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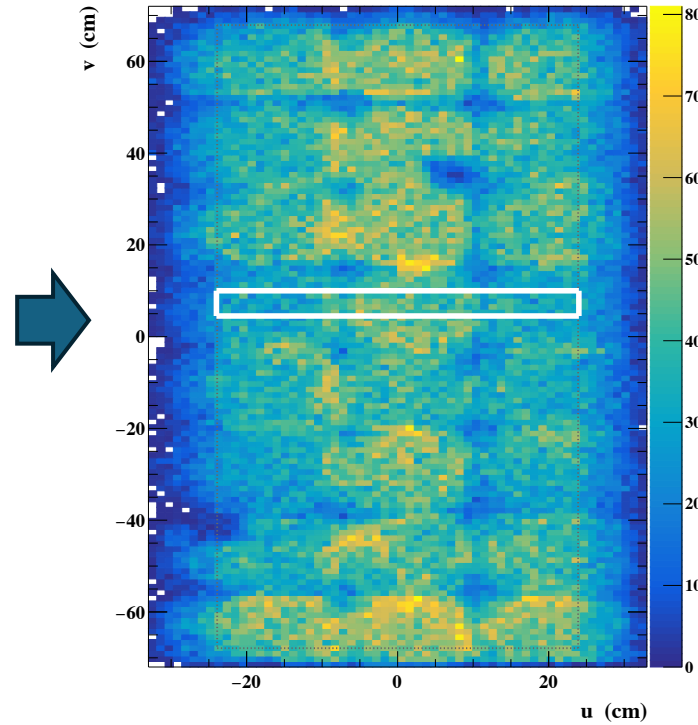
1. L2 regularization ($\mathbf{A} \rightarrow \mathbf{A} + \lambda \mathbf{I}$):
increase determinant to stabilize the solution
2. Geometrical smoothness ($\mathbf{A} \rightarrow \mathbf{A} + \mu \mathbf{L}$, $L_{i,j} = 1$ if (i, j) is neighbor, else 0):
penalty term $(w_i - w_j)^2$ for the neighboring sensor.
3. Non-negative weights:
if $w_i < 0$ is obtained, cap w_i to 0 and solve the system again.

Segmentation

Training region

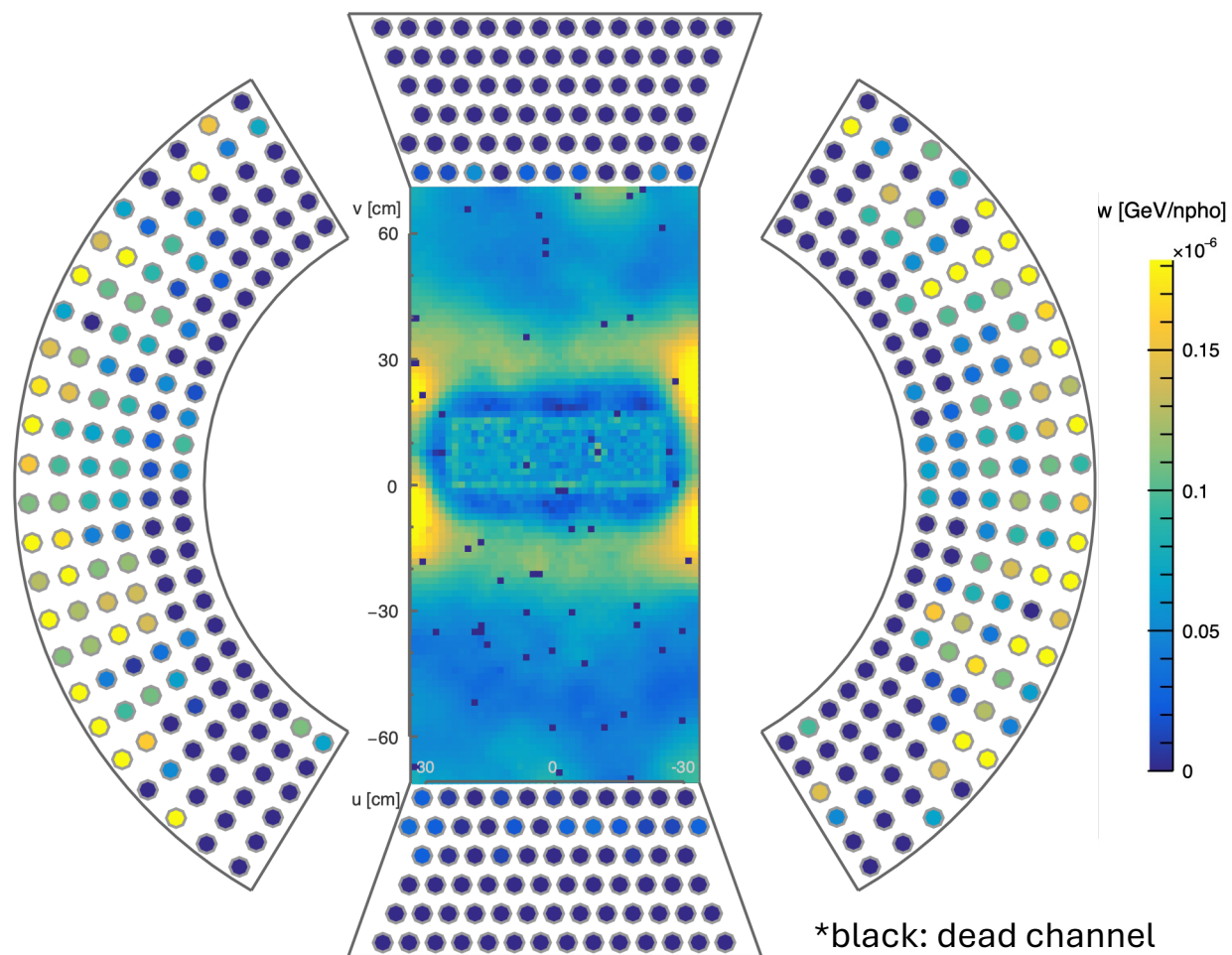


Application region



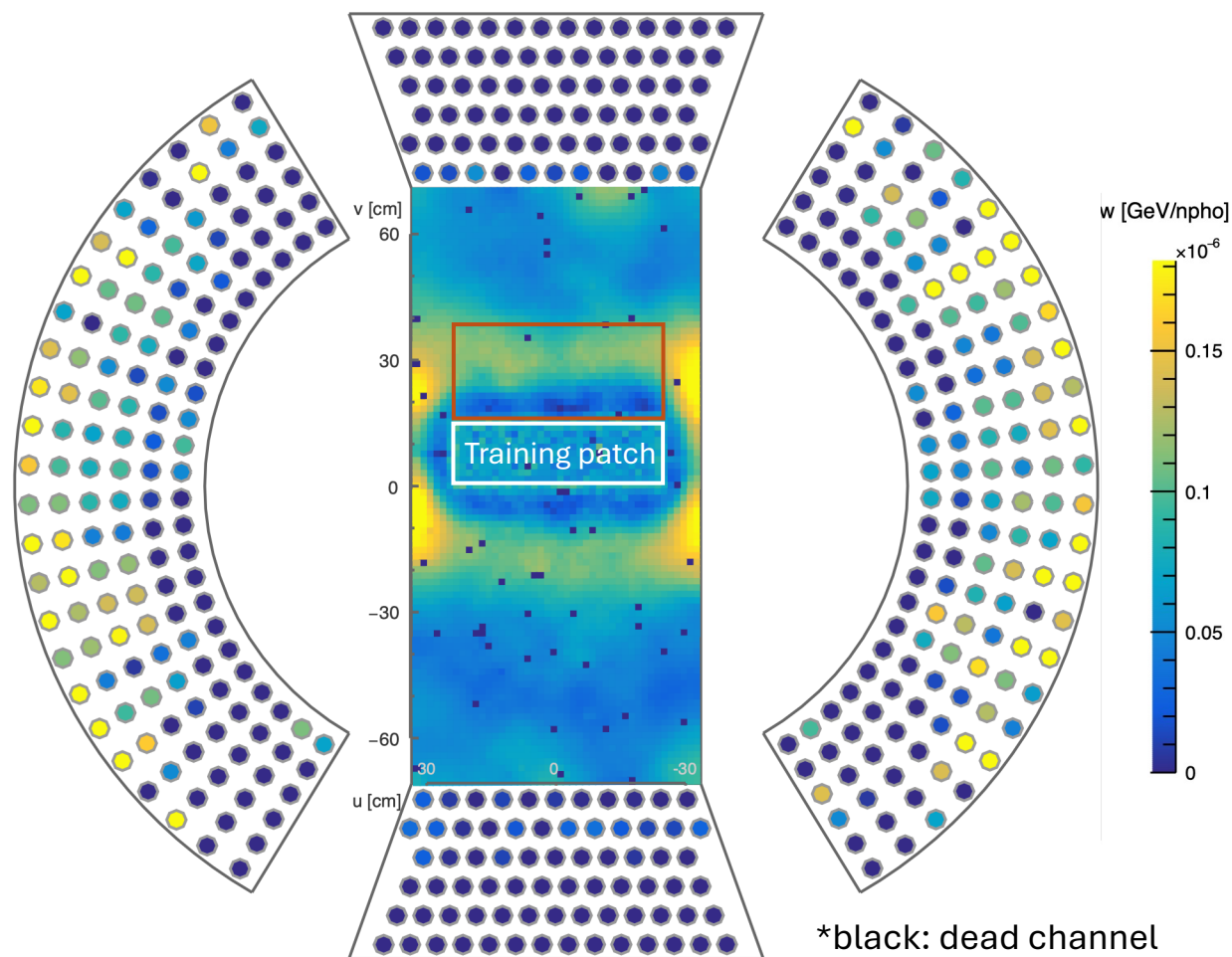
- Segmented in 48 patches (24 in v , 2 in w) depending on the reconstructed position of γ → as fine as possible while not to overfit
- Margin between training and application patch, preventing edge effect (next page)
- Training/Validation split: 80/20

Weight map



A pattern correlated to training sample observed

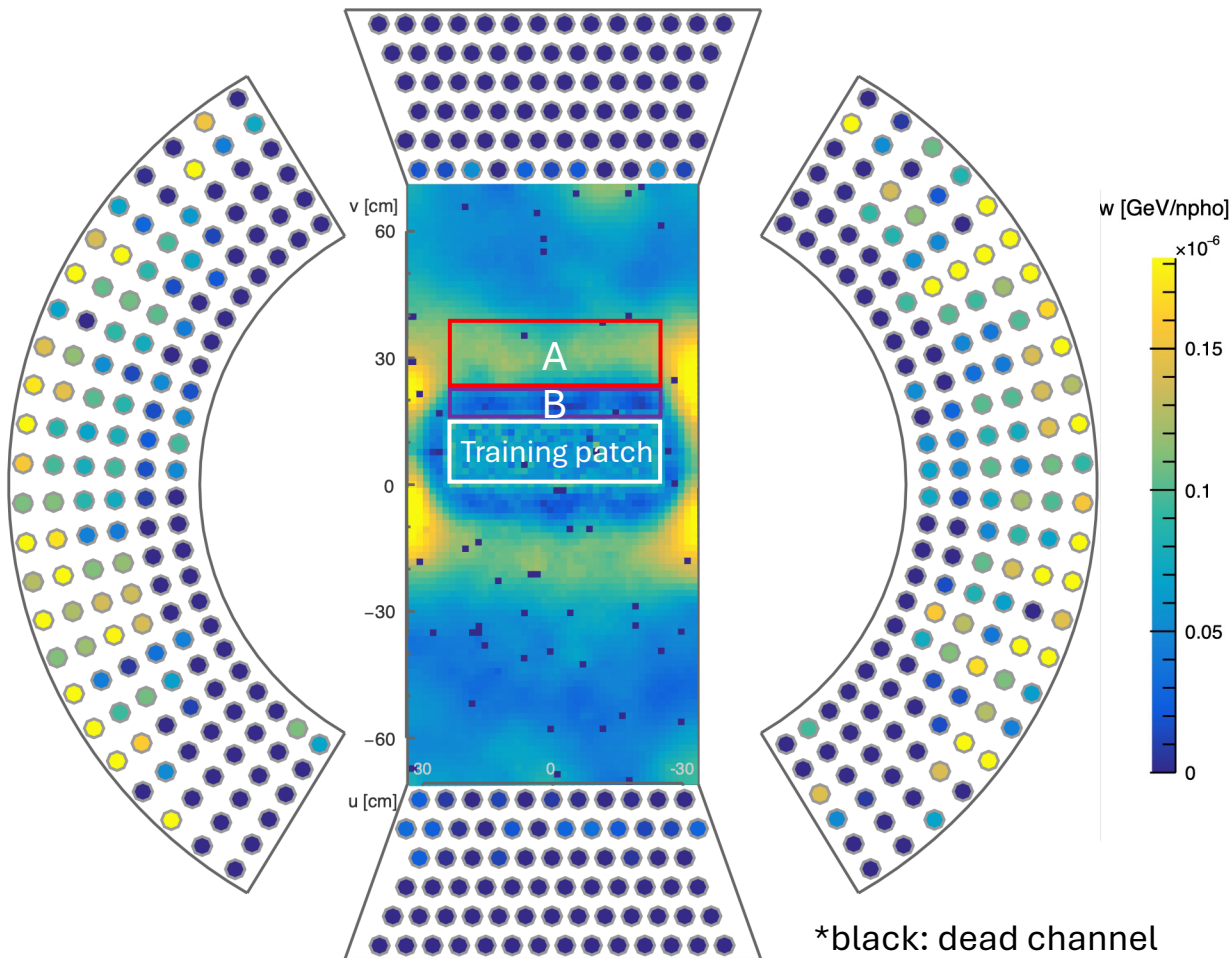
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A pattern correlated to training sample observed

- Sensors outside the training patch have positive correlation in detected #photons
→ Without regularizations, negative weights emerge by nature

Weight map



A pattern correlated to training sample observed

- Sensors outside the training patch have positive correlation in detected #photons
→ Without regularizations, negative weights emerge by nature
- More detected #photons variation in region B
 - Due to geometrical effect etc...
 - **Not photoelectron statistic concerning the weight distribution**
→ Smaller weights for region B

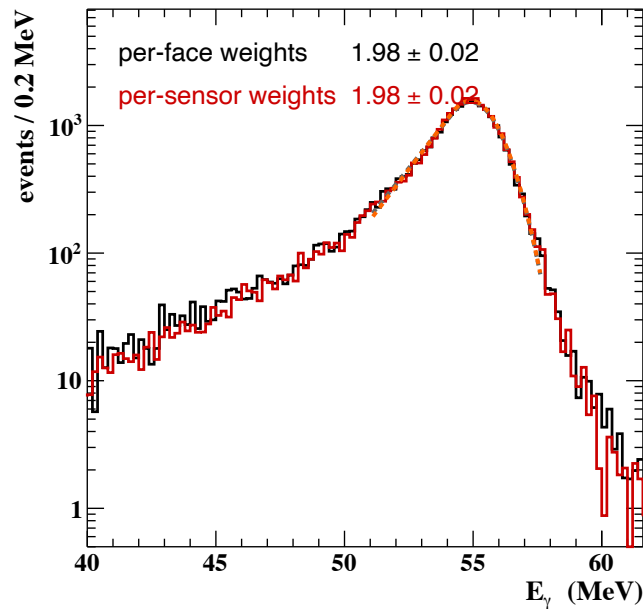
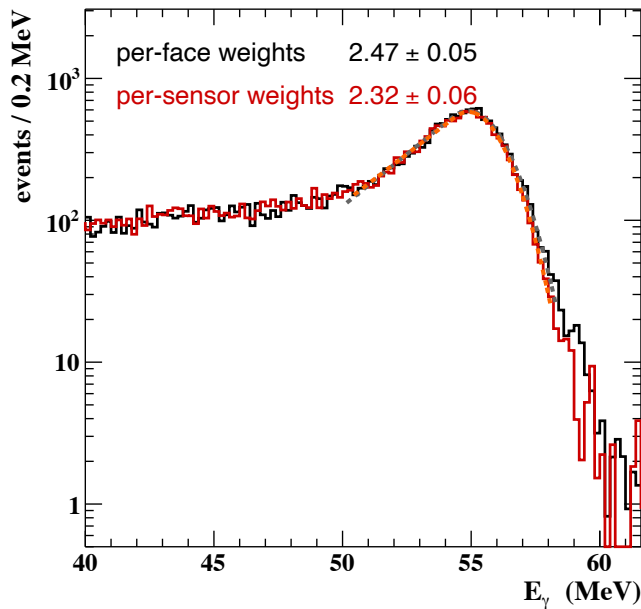
Energy spectrum

Reconstructed 55 MeV spectrum fitted with ExpGaus.
Resolution = σ/μ

$$\text{ExpGaus} = \begin{cases} A \cdot \exp\left[-\frac{1}{2}\left(\frac{E-\mu}{\sigma}\right)^2\right], & E > \mu + t \\ A \cdot \exp\left[t\left(\frac{t}{2} - E + t\right)/\sigma^2\right], & E > \mu - t \end{cases}$$

$w < 2$ cm

$w > 2$ cm



Improvement in shallow region (=closer to SiPMs)
 $2.47\% \rightarrow 2.32\%$

SiPMs has much higher granularity but had the same weight for the entire face
→ Benefit from the per-sensor weighting

Summary

- E_γ resolution is crucial for the sensitivity of the MEG II experiment.
- A new sensor weighting method for energy reconstruction is developed.
- Least square fit with regulations to obtain sensor-by-sensor weights.
- Improves resolution in shallow region ($w < 2$ cm): 2.47% $\rightarrow$ 2.32%
- Aiming to implement for the next MEG II result.